Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
E		**2224327**

Forestry for Carbon Sequestration to Forest Management

Summary

It is well known that forests have strong carbon sequestration value. In addition, forests have ecological, recreational value, and so on. We need to develop optimal forest management plans based on its composition, local climate and other characteristics, combined with its values.

Firstly, we establish a logistic equation for tree accumulation So with respect to tree age of different trees. According to tree accumulation, we calculate the carbon sequestration of the forest by the Accumulation Method(IPCC Method). For forest products made from trees, we model the exponential decay of carbon sequestration and classify them into four categories. We introduce processing and transportation losses α, which is simulated in between 0.2 and 0.5 using the Monte Carlo method. At last, we establish a single-objective planning model and conduct traversal of the cutting rate to obtain the management plan for the forest, using the total carbon sequestration C_{total} as the objective function and the cutting rate Cut_s as the decision variable. The result shows that the best harvesting rate for common forests is 0.05-0.01, with 10-20 years rotation period.

Secondly, we assess the integrated value of the forest, subsume the carbon sequestration value into the protection value,and measure each expressed value quantitatively. We introduce a maximum sacrifice factor $\delta_i(i = 1, 2, 3)$ for each value to ensure a relative balance among the values. We classify the forests into two major categories, non-commercial forests and non-wood product forests, which include a total of five sub-categories. For some unique forests, we do not cut them down under legal, economic or specific environmental restrictions, such as nature reserve forests and mother forests. Since different types of forests have different growth rates, there are transition points between management plans. In addition, we introduce an environmental growth factor $\gamma(0.6 - 1.3)$ according to the environment and location of the forest. At last, we take the comprehensive value of forest as the target and the cutting rate as the decision variable, to obtain the selective felling cycles of fast growing forest, common forest and ecological forest as 5-10 years, 20-40 years and more than 70 years, respectively.

Then, we take the data of penobscot experimental forest in Maine, and put it into model II to calculate the carbon sequestration. We find that the optimal cutting rate of this forest is 0.048, that is, the forest cycle is 20.8 years and stable carbon storage for 100 years is $4.858 \times 10^{10} tC$ and the CO_2 sequestration is $1.78 \times 10^6 t$. There are different transitional management plans for different types of forests and different needs of forest managers.In general,we divide the transition strategy into two approaches, namely, applying for planting to expand the area and switching to a round of trees with a short growth cycle.

Through the research, we believe that the forest managers should include logging management plan. We write a non-technical article to persuade the local community to conduct logging management.

Keywords: Carbon Sequestration, Logistic Equation, Forest Value Assessment, Single-objective Planning, Forest Management, IPCC Method

Contents

1 Introduction and Assumptions

1.1 Background

Wildfires that ravaged the western United States in 2020 set new records, highlighting the consequences of global warming and habitat degradation. As a result, we must lower the amount of carbon dioxide in the atmosphere to mitigate the effects of climate change. We may use photosynthesis to absorb carbon dioxide in the atmosphere in plants, soil, and water, a process known as "carbon sequestration," in addition to reducing greenhouse gas emissions. Forests are the earth's terrestrial ecosystem with 56% of the entire terrestrial carbon pool, and have great value in carbon sequestration. Carbon sequestration in the forest is shown in Figure 1.

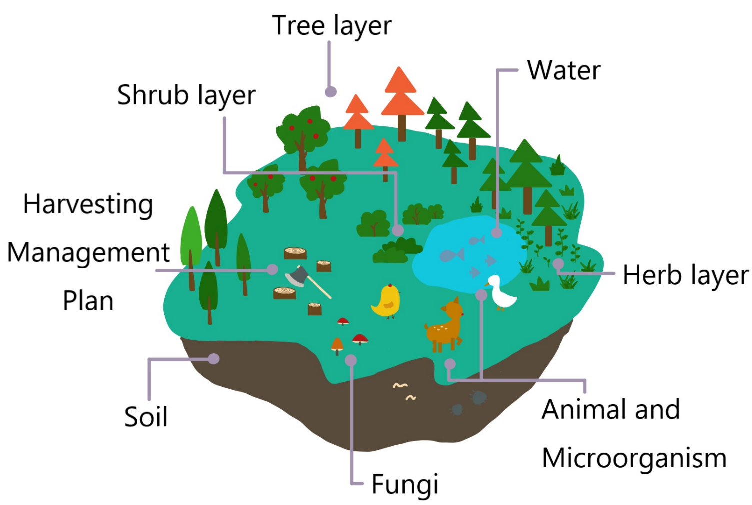

Figure 1: Diagram of carbon sequestration in forests

Forest trees can be harvested for products, and these forest products can sequester carbon dioxide over their lifetimes. Numerous studies have found that the carbon sequestration of various forest products, when combined with the carbon absorbed by young forest regeneration, has the potential to sequester more carbon than uncutting forest in the same time.As a result, cutting down trees will be a key component of the forest management strategy.

What we want to do is to simulate the process of carbon sequestration from forest growth to cutting to products, and explore the balance of the three to maximize the benefits of the forest.

1.2 Restatement of the Problem

The problems to be solved are as follows:

(1) A carbon sequestration model is developed to calculate the amount of carbon dioxide sequestered by the forest and its products over a certain period of time, and to determine the most effective forest carbon sequestration management plan based on this.

(2) Consider other values of the forest and develop a forest management plan that is most beneficial and advantageous to society. The model should identify a forest management plan that balances the various ways in which the forest is assessed, including the boundaries

of the decision model, the conditions for cutting down the forest, the transition points that apply to all forest plans, and how the transition points are determined using specific forest characteristics.

(3) Using a forest as a case study, the decision model is used to obtain a management plan that includes harvesting, to find and demonstrate the optimality of the amount of CO_2 sequestered in this forest over 100 years using this management plan, and to discuss transition strategies for forest management when the plan is changed.

(4) Write a popular newspaper article to inform the local community that logging is an important part of the optimal forest management plan.

1.3 Assumptions

To simplify the problem and make it convenient for us to simulate real-life conditions, we make the following basic assumptions, each of which is properly justified.

- **Assumption 1:** It is assumed that the "carbon sequestration forest" only considers the living plants and soil in the forest, i.e., the carbon sequestration effect of dead plants, animals, microorganisms and water in the forest is not considered, and all plants and their processed products can fully utilize their carbon sequestration effect.
- **Assumption 2:** Among the plant conditions for carbon sequestration, the life cycle of plants other than perennial trees is short, and the change of year cycle has little effect on the overall carbon sequestration. Thus, we only consider the effect of perennial trees on carbon sequestration in the plant category. The life cycle of the final product obtained from raw materials is positively related to wood quality.
- **Assumption 3:** The life cycle of the final product obtained from raw materials is positively related to wood quality.
- **Assumption 4:** It is assumed that all trees in the forest are in the same current state of growth at the time of the BMP(Best management plan), and that all timber in the forest discussed in question 1 is available for harvesting, i.e., there are no legal or other factors that make it unavailable for harvesting.
- **Assumption 5:** The modeling process assumes that there are no natural disasters in the forest and that all trees are growing under suitable conditions.

2 Notations

Symbols	Description	Unit
n	Total number of tree species in the forest	-
N_i	Number of trees of the i-th species	-
$C_{tree,i}$	Carbon stocks of the i-th tree species	tC
Bi_i	Average biomass per unit area of the i-th tree species	$t.d.m/hm^2$
CF_i	Carbon content ratio of the i-th tree species	$tC/t.d.m$
S	Area of tree stands	hm^2
St_i	Accumulation per unit area of the i-th tree species	m^3/hm^2
SVD_i	Basic wood density of the i-th tree species	$t.d.m/m^3$
BEF	Biomass expansion factor of the i-th tree species	-
So_i	Single plant accumulation of i-th tree species	$t.d.m$
$C_{live,tree}$	Total amount of carbon sequestered by trees	tC
LS_i	Lifespan of the i-th tree species	year
α	Loss rate of forest products due to processing and transportation	-
C_{total}	Total carbon sequestration	tC
C_{CO2}	Total dioxide sequestration	t
P_c	Value of dioxide sequestration per ton	$\$/t$
γ	Environmental growth factor	-
Rev_i	The i-th value of the forest	$
$Value_{total}$	Total value of the forest	$
δ_i	Maximum sacrifice factor of i-th value of the forest	-

where we define the main parameters while specific value of those parameters will be given later.

3 Model Overview

In dealing with the first question, we develop a single-objective planning model with the maximum total carbon sequestration C_{total} as the objective and the cutting rate Cut_s as the decision variable. The total carbon sequestration is the sum of forest carbon sequestration and forest product carbon sequestration. We conduct traversal of the cutting rate to obtain the management plan for the forest.

In dealing with the second question, we model the integrated value of the forest, including both direct and indirect values. Among them, the direct value includes economic value, and the indirect value includes ecological value and social value. We subsume the carbon sequestration value into the ecological value. Then, we establish a single-objective planning model with the objective of maximizing the integrated forest value $Value_{total}$ and the cutting rate Cut_s as the decision variable. Similarly,we conduct traversal of the cutting rate to obtain the management plan for the forest.

In dealing with the third question, we use data from the Penobscot Experimental Forest in Maine, bring it into the model II, and perform carbon sequestration calculations to obtain the optimal cutting rate of this forest and carbon dioxide sequestration by this forest and its products over 100 years.

4 Model I: Carbon sequestration model and management plans in forest

Forests are capable of absorbing and fixing carbon dioxide from the atmosphere, and their forest products are also capable of carbon sequestration, as model in Figure 2. To better manage the forest, we modeled the carbon sequestration of various trees and their products over time to determine the best harvesting plan to manage the forest and to ensure that the program maximizes CO_2 sequestration.

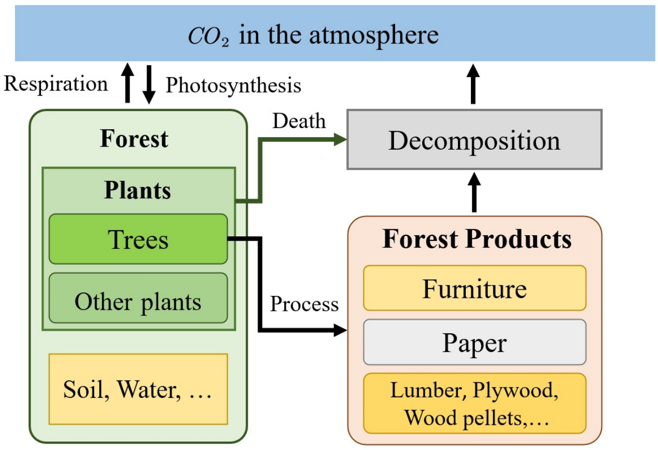

Figure 2: Carbon dioxide fixation by forests and their products

4.1 Carbon sequestration model

4.1.1 Carbon sequestration of trees

The stock approach is a carbon estimation method based on forest stock data. We assume that there are n species of trees in a forest, and according to the literature [**6**], Carbon stocks of the i-th tree species have the following equation.

$$C_{tree,i} = Bi_i \, CF_i \, S \tag{1}$$

where, $C_{tree,i}$ is carbon stocks of the i-th tree species,B_i is average biomass per unit area of the i-th tree species,CF_i is carbon content ratio of the i-th tree species$(i = 1, 2, ..., n)$,S is area of tree stands. The Bi_i in equation (1) can be obtained by choosing the following:

(1) For tree species for which industry standards for "standing wood biomass models and carbon measurement parameters" have been published, the following equations are used based on the data obtained from the carbon pool survey for each tree species measurement factor.

$$Bi_i = f_i(x_{i,1}, x_{i,2}, x_{i,3}, ...) \tag{2}$$

where $f_i(x_{i,1}, x_{i,2}, x_{i,3}, ...)$ translates the measured tree factor into a regression equation for above-ground biomass.

(2) For tree species for which industry standards for "standing wood biomass model and carbon measurement parameters" have not been published, the following formulae are used for each species' unit area storage volume, basic wood density of the species and biomass expansion factor obtained from forest ecosystem carbon pool surveys and measurements.

$$Bi_i = St_i \times SVD_i \times BEF \tag{3}$$

where, St_i is accumulation per unit area of the i-th tree species, SVD_i is basic wood density of the i-th tree species, BEF is biomass expansion factor of the i-th tree species($i = 1, 2, ..., n$). We denote single plant accumulation of i-th tree species as So_i,and total number of trees of the i-th species as N_i.So,we have

$$St_i = \frac{So_i N_i}{S}. \tag{4}$$

In the above equation, we can find CF_i, SVD_i for different trees, and BEF and So_i for different trees at different ages by looking up the table. We write down the cutting rate as Cut_s, and we have.

$$N_i = N_{i,0} \times Cut_s \tag{5}$$

where $N_{i,0}$ denotes the number of trees of that species in the previous year.

Although the calculation of carbon sequestration of trees is measured in terms of age, i.e., its carbon sequestration function is actually *discrete*, the stumpage accumulation So_i for trees is related to the growth of trees and can be considered as a *continuous process* as shown in Figure and is consistent with the logistic equation:

$$\frac{dSo_i}{dt} = \frac{rSo_i(K - So_i)}{K} \tag{6}$$

that is

$$So_i = \frac{KAe^{rt}}{1 + Ae^{rt}} \tag{7}$$

where t is the time, r is the growth potential index of stumpage So_i, and K is the maximum possible accumulation.

For different trees, we know the BEF and So_i at five ages: young, intermediate, near-mature, mature and over-mature, divide the different ages of different tree species into the corresponding five stages, and fit their data using the least squares method to obtain BEF and So_i as a function of the age time t, that is:

$$BEF = f_1(t),\ So_i = f_2(t).$$

Then, $C_{tree,i}$ is also a function of and tree age time t.

In summary, for trees of age t in a given forest, the total amount of carbon sequestered by n species of trees $C_{live,tree}$ is:

$$C_{live,tree}(t) = \sum_{i=1}^{n} C_{tree,i}(t). \tag{8}$$

4.1.2 Carbon sequestration of forest soil

Soil carbon fixation is also an important way of carbon sequestration. Soil carbon sinks include organic and inorganic carbon sinks, the former being the conversion of plant organic matter into organic carbon in mineral soils by decomposition, and the latter being the conversion of atmospheric carbon dioxide into primary or secondary minerals in the soil. The Soil Institute of America defines soil carbon sequestration as the direct or indirect storage of atmospheric carbon dioxide in the form of stable solids in the soil, including the direct conversion of carbon dioxide into soil inorganic matter such as calcium or magnesium carbonate, or the indirect conversion of atmospheric carbon dioxide into plant energy through plant photosynthesis, which is fixed as soil organic carbon in the decomposition process.

According to the literature[**6**], we obtained the soil carbon stock in the carbon pool of the forest ecosystem carbon stock as:

$$C_{soil} = SOCC \times S \tag{9}$$

where, $SOCC$ is the soil organic carbon density of forest stands, which can be obtained from the data by looking up the table of different types of forests, S is the area of forest stands.

4.1.3 Carbon sequestration of forest products

According to **Assumption 1**, when considering forest products, we can only consider the effect of perennial tree products on carbon sequestration, i.e., only woody forest products. Thus, we classify wood forest products into four categories, including: lumber, plywood, paper and other production, as shown in Figure 3.

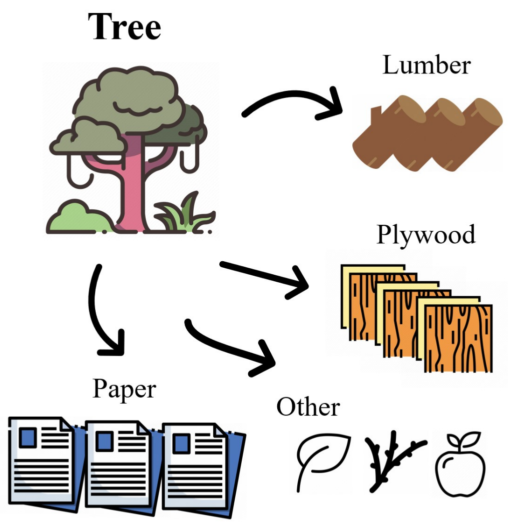

Figure 3: Classification of woody forest products

Woody forest products are an extension of the carbon sequestration function of forests. The carbon storage capacity of woody forest products refers to the amount of carbon stored in woody forest products during their life cycle. The biomass contained in the trees in a forest is converted to woody forest products after harvesting, and some of the absorbed carbon continues to be stored in the product. The amount of carbon sequestered declines each year of the product's

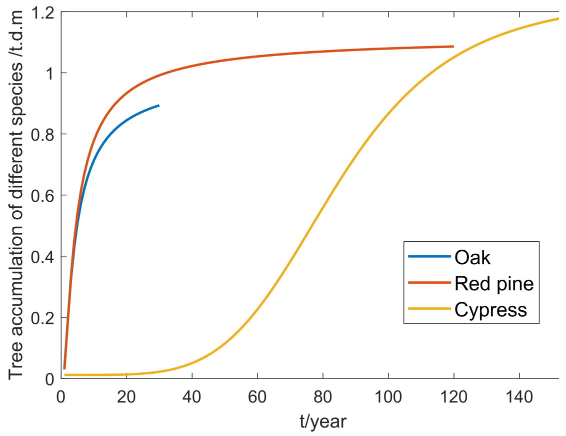

Figure 4: Individual accumulation of species

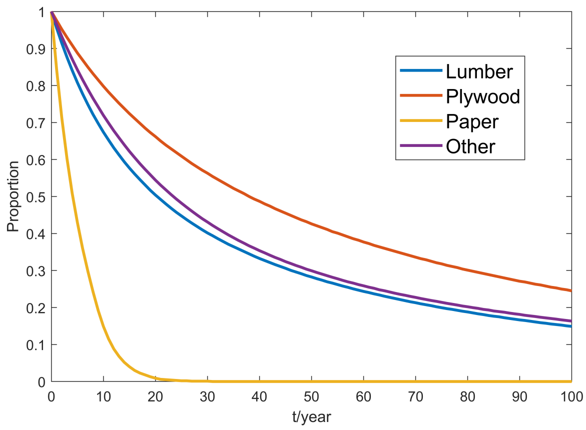

Figure 5: Product decay curves

life, and the product is eventually discarded, buried, or burned. The decay of the four types of woody forest products over time is shown in Figure 5.

The exponential decay model of carbon sequestration of the corresponding products is obtained by fitting the curves with the least squares method

$$De_k = b_{k,0} \cdot e^{b_{k,1}t}.$$

According to international studies [**8**], woody fuels account for nearly 50% of global logging consumption, for which we approximate that only 50% of a tree can be converted into relevant forest products, i.e., the carbon sequestration of forest products is only 50% of the carbon sequestration of the original trees, but because woody products have a long service life, they can be used as an additional way of carbon sequestration. In addition, due to the process of processing and transportation, forest products will also have losses, and we record the loss rate generated in this process as α. We use Monte Carlo method to simulate α stochastically in the range of 0.2-0.5.

Then the carbon sequestration $C_{i,k}$ of the i-th tree at age t_0, produced into the k-th product at year t, is

$$C_{i,k}(t) = \frac{\alpha\, De_k(t)\, C_{tree,i}(t_0)}{2}. \tag{10}$$

Thereby, the total carbon sequestration $C_{i,k}$ of the i-th tree at age t_0 to produce the four categories of products at year t is

$$C_{pro,i}(t) = \sum_{k=1}^{4} \beta_k\, C_{i,k}(t). \tag{11}$$

where β_k denotes the proportion of forest products of that species made into the k-th category and satisfies $\sum_{k=1}^{4} = 1$.

4.1.4 Total carbon sequestration and CO_2 sequestration

Based on the carbon sequestration models for soil, trees and woody forest products, we obtain the total forest carbon sequestration C_{total} as:

$$C_{total} = C_{soil} + \sum_{t=1}^{LS} + \sum_{i=1}^{n} \sum_{t=1}^{LS} C_{pro,i}(t). \tag{12}$$

Since 1 ton of carbon is equivalent to 3.667 tons of CO_2, the carbon sequestration is converted to CO_2 sequestration by the following equation.

$$C_{CO2} = 3.667 C_{total}. \tag{13}$$

where C_{CO2} is the CO_2 sequestration amount in tons.

4.2 Management plans in forest

Distinguish different forest management practices by selective forest cutting ratio

Since the i-th tree species corresponds to the production of m_i products, there are at most $\sum_{i=1}^{n} m_i$ theoretically optimal cutting times for different products of different trees for n tree species of the forest. Although we simplified the product categories by dividing them into four major categories in total, there is no doubt that this is still quite difficult for practical forest management. Therefore, we seek to find an equilibrium point in the optimal cutting time of all trees, so that this equilibrium point can make the whole forest reach the maximum in carbon sequestration.

Management model

In this problem, we only consider the carbon sequestration effect of the forest and its woody forest products. We need to adopt an appropriate forest management approach, i.e., to determine the cutting rate Cut_s for the forest so that the forest itself and its woody forest products sequester the maximum amount of carbon to achieve the maximum benefit.

Then the objective function is:

$$\max \quad C_{total} = C_{soil} + \sum_{t=1}^{LS} + \sum_{i=1}^{n} \sum_{t=1}^{LS} C_{pro,i}(t) \tag{14}$$

s.t. to equation (1)-(12).

4.3 Analysis of Result

To obtain the best carbon sequestration management for a forest, we first need to determine the best time to cut down the trees. We know the carbon sequestration curve of a certain tree and the decay curve of the carbon storage of a certain product produced. By determining the location of a given tree, we can obtain the amount of carbon sequestration produced at that location, shown as Figure 6.

The growth curve of a certain color is the carbon content curve of the tree growth process, and the decrease curve is the decay curve of the carbon content of the product, and the combination of the two forms the carbon storage cycle of a tree. Obviously, for a cycle, maintaining growth will always result in the highest carbon sequestration. Thus, for a given location in the forest, the maximum carbon sequestration it can produce is determined by the sum of the carbon content of the currently growing tree and the carbon content of its previous cycles, i.e., the sum of all curves at a given point in the graph .

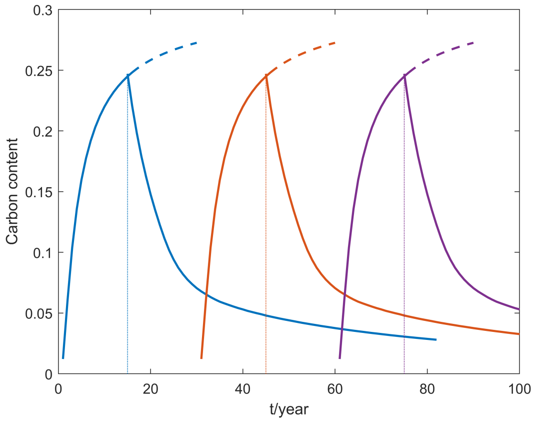

Figure 6: The carbon sequestration of growth and the decay of specific products

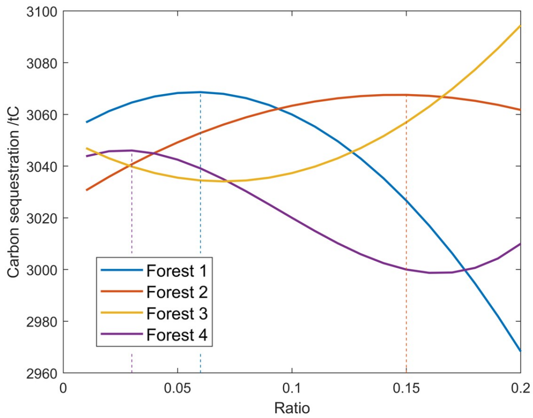

Figure 7: Carbon sequestration curves of forests at different deforestation rates

The carbon content at this location at this time is equal to the sum of the carbon content of the currently growing tree and the carbon content of the products of the previous two cycles, as shown by the peak point of the purple line in the figure. We know the carbon sequestration curves of the various tree growths and the decay curves of the carbon content of the various products they produce, and sum the carbon content of each cycle on the time axis, with long enough time iterations to obtain its stable maximum value. From this, it is possible to determine the optimal time to cut a certain species of trees when they are used to produce a certain product.

It is worth mentioning that in our model, there are multiple trees in a forest, each with multiple products, and different products have different carbon decay curves, so we can eventually get a series of optimal cutting times.

After determining the optimal cutting time, we seek the optimal point of the harvesting management model by adjusting the cutting ratio - that is, the cutting ratio at which each tree species can achieve the maximum amount of carbon storage. The program iterates through the cutting ratios to find the optimal cutting time for each species. After inputting the data into the Forest Harvesting Model, the amount of carbon storage for each species at a given point in time is summed to obtain the stable carbon storage at that harvest rate. Finally, by comparing the carbon storage of all cutting rates, the optimal cutting rate is found, and then the optimal harvesting management model is developed, Figure 7 shows the carbon sequestration curves of various forests under different cutting rates.

Finally, we can get three types of forest management plans (shown in Figure 7): one is fast-growing forest, as shown in Figure Forest 3, with the optimal cutting rate greater than 0.2, which means the rotation period is less than 5 years; one is normal forest, as shown in Figure Forest 1, with the cutting rate between 0.1 and 0.05, and the rotation period is 10-20 years; and one is ecological forest, as shown in Figure Forest 1, with the cutting rate less than 0.05, which means the rotation period is less than 20 years. which means the rotation period is greater than 20 years. Also a mixture of various forests may produce Forest 4 images, making it possible to have multiple extreme value points within the appropriate cutting rate range.

5 Model II: Integrated forest value evaluation model and management plan

5.1 Integrated forest value evaluation model

In the first question, we set deforestation rates for forest management plans based only on the amount of carbon sequestered in the forest. However, there are many other aspects of forest values, and the value of the forest in terms of carbon sequestration is only part of the ecological value of the forest. The forest management plan that is most favorable to carbon sequestration is not necessarily the most favorable to the overall value of the forest.

In order to better develop forest management plans based on the overall value of the forest, we first construct an integrated forest value evaluation model.

The value of the forest includes direct value and indirect value, where the direct value is economic value and the indirect value is ecological value and social value. The comprehensive value of forest is shown in Figure 8.

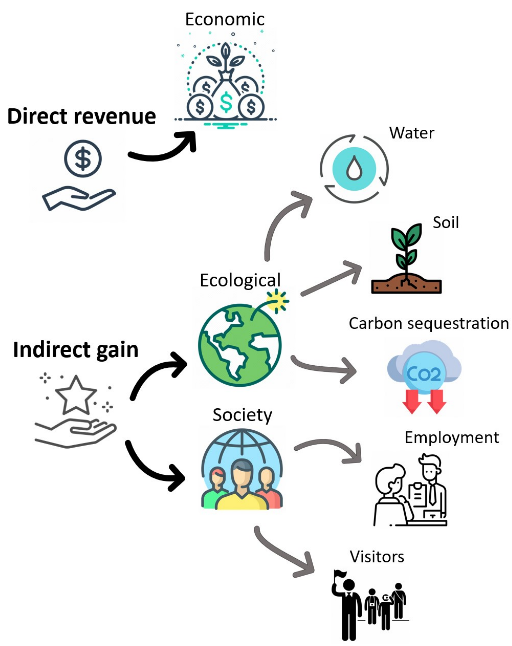

Figure 8: Classification of woody forest products

5.1.1 Direct revenue: Economic value

The direct benefit is the economic value of the timber directly obtained after the forest is harvested. It is known that for a certain tree i, the stock of a certain tree of age t_0 is $So_i(t_0)$, and the loss rate when transporting and processing forest products is α. We note that the average value of the species is M_i in $\$/m^3$, and the number of trees of age t_0 cut by the species is Cut_{i,t_0}, then the direct economic benefit Rev_1 obtained at a certain harvest management of the forest is.

$$\sum_{i=1}^{n}\sum_{t_0=1}^{LS_i} \alpha So_i(t_0) Cut_{i,t_0} M_i. \tag{15}$$

5.1.2 Indirect gain

For the indirect values of forests, we classify them into two categories: ecological values and social values. Ecological values include the value of water, soil, and carbon dioxide sequestration, and social values are the contribution of forest development to tourism and employment.

Each value of **Ecological values** is expressed as follows:

- **Water source protection value**:

  Forest soil has good structure, fast infiltration and high water storage, which can well regulate the slope runoff. Its water source protection value $Rev_{2,1}$ is:

$$Rev_{2,1} = QP_w \tag{16}$$

  where, P_w is the water price, Q is the amount of water contained in the forest, and by regression method we have:

$$\ln Q = -2.16 + 0.540(1 - Rt_{young}) \ln S + 0.728 \ln(Pc - Ec) \tag{17}$$

  where, Rt_young is the proportion of young trees in the forest, and Pc, Ec is the annual rainfall and evapotranspiration of the area where the forest is located, respectively. The regression coefficients and significance analysis are shown in Figure .

	coefficient	t	Sig.
constant	-2.160	-1.028	0.362
$\ln(Pc - Ec)$	0.728	5.503	0.005
$(1 - Rt_{young}) \ln S$	0.540	2.204	0.092

- **Soil protection value**:

  Forests have a high soil protection value because they can prevent the loss of soil and its nutrients, prevent and control sand, and reduce soil salinization. Its soil protection value $Rev_{2,2}$ is:

$$Rev_{2,2} = A_s P_s = \frac{\mu S}{\rho H_s} P_s \tag{18}$$

  where, A_s is the annual area of abandoned land reduced by the forest (hm^2), P_s is the average gain of forestry production $(\$/hm^2)$, μ is the soil erosion modulus, ρ is the soil capacity, which is taken as $1.3t/m^3$, H_s the average forest soil thickness (m).

- **Carbon dioxide fixed value**:

  Take the Swedish carbon tax rate method, which is commonly used internationally, to calculate the value of carbon dioxide fixed by the forest, with the following formula

$$Rev_{2,3} = P_c C_{Co_2} \tag{19}$$

where C_{Co_2} is the total Co_2 sequestered in the forest and P_c is the fixed value per ton of CO_2 of $150\$/tC$.

Different trees located in different regions have different growth rates. For example, the lower the forest climate located at high latitudes, the slower the growth rate of trees, the smaller the growth rate of tree accumulation; the higher the forest climate located at low latitudes, the faster the growth rate of trees, the larger the growth rate of tree accumulation. For this, we introduce an environmental growth factor influence coefficient γ, which is between 0.6 and 1.3, and the faster the trees grow, the larger the γ. The is modified to obtain the following equation.

$$\frac{dSo_i}{dt} = \frac{\gamma\, rSo_i(K - So_i)}{K} \tag{20}$$

that is

$$So_i = \frac{KAe^{\gamma\, rt}}{1 + Ae^{\gamma\, rt}}. \tag{21}$$

So, we identify the temperature of the environment in which the forest is located by using its location characteristics. Temperature can have an impact on the growth rate of trees and thus on the Carbon stocks of the i-th tree species, i.e., $C_{tree,i}$.

From the above analysis, we obtain the ecological value of the forest Rev_2 as the sum of the above three values, i.e.

$$Rev_2 = \sum_{i=1}^{3} Rev_{2,i} \tag{22}$$

Then, we discuss **Society values**.

- **Recreational values**:

  According to the paper [**9**], we obtain three stages of the Recreational values evaluation function P_{rec} (units have been converted to dollars). The forest in question has an area of 100 hectares and a visitor density of about 10 people per hectare per year, of which 50% should be willing to pay the associated costs.

$$Rev_{3,1} = \begin{cases} 4,442.70 & ,50 \leqslant T < 120 \\ 7.3783T + 3557.30 & ,120 \leqslant T < 180 \\ 6,496.25 & ,T \geqslant 180 \end{cases} \tag{23}$$

  where, T denotes the Rotation period and the function image is shown in Figure 9:

- **Employment value**:

  The forest protection and harvesting management requires technicians and managers, and thus can contribute to employment to some extent. Then the employment value $Rev_{3,2}$ is:

$$Rev_{3,2} = P_{wages}\, Peo_s \tag{24}$$

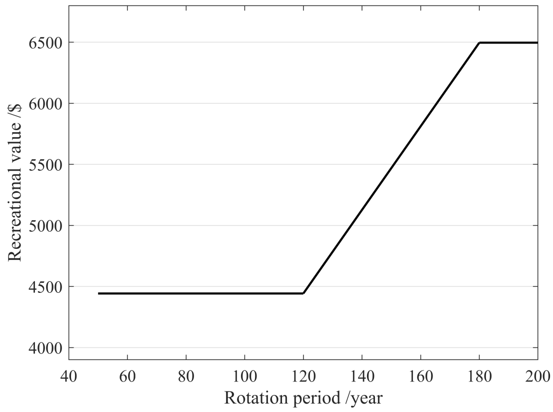

Figure 9: Function image of recreational value

where P_{wages} is the annual salary of employees, which can be expressed using the GDP per capita of the region, Peo_s is the number of jobs provided by the forest, and:

$$Peo_s = ks_1 Cut_s S + ks_2 S. \tag{25}$$

where Cut_s is the harvesting rate of this forest, S is the area of this forest,ks_1 is the employment coefficient of the forest harvesting industry, and ks_2 is the employment coefficient of the forest area.

Then the social value of the forest Rev_3 is the sum of the above two values, i.e.

$$Rev_3 = \sum_{i=1}^{2} Rev_{3,i}. \tag{26}$$

In summary, the total value of the forest $Value_{total}$ is the sum of its economic, ecological and social values, i.e.

$$Value_{total} = \sum_{i=1}^{3} Rev_i. \tag{27}$$

5.2 Balance of forest values considering sustainability

The value of the forest in terms of carbon sequestration is only part of the ecological value of the forest, and the forest management plan that is most favorable to carbon sequestration is not necessarily the most favorable to the overall value of the forest. When considering the overall value of a forest, the individual values are mutually constraining. For example, to obtain more economic value Rev_1, the rate of tree cutting Cut_i should be increased as much as possible, but an increase in the rate of cutting will result in a decrease in the ecological value of the forest Rev_2. So there is a constraint relationship between the values.

Considering sustainable development, i.e., about the coordinated development of nature, economy, and society, economic growth that focuses on long-term and lasting development. We need to ensure the stable existence of forest ecological as well as social values, not to obtain temporary economic development at the expense of the environment, and to take into account the carrying capacity of the environment. In this regard, we introduce corresponding sacrifice coefficients δ_i for each value, and record the evaluated value of each value at the beginning as $Rev_{i,0}(i = 1, 2, 3)$, then for each deforestation the forest value Rev_i has:

$$Rev_i \geq \delta_i Rev_{i,0}. \tag{28}$$

For different forests, the corresponding sacrifice coefficients δ_i are introduced for each value.

We first classify the forests as shown in Figure .

We classify the forest into two parts, non-commercial forest and commercial forest, shown as Figure 10. Among them, non-commercial forests include shelter forests and special-purpose forests. Shelter forests consist of water connotation forests, soil and water conservation forests, etc. Special-purpose forests include defense forests, experimental forests, mother forests, nature reserve forests, etc. Commercial forests include timber forests, firewood forests, and non-wood product forests.

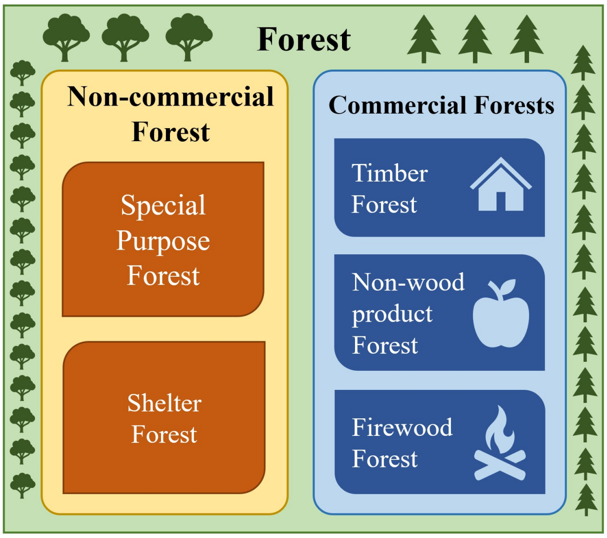

Figure 10: Diagram of forest standard classification

Generally speaking, the sacrifice coefficients δ_2, δ_3 are higher for ecological and social values, and the economic values generated by logging can be lower, even the sacrifice coefficient δ_1 can be 0. In contrast, the sacrifice coefficients δ_2, δ_3 are lower for economic forests, which do not need to maintain high ecological and social values, and the sacrifice coefficients δ_1 are higher for economic forests. δ_2, δ_3 is lower, while the sacrifice coefficient δ_1 corresponding to the economic value is higher.

5.3 Management plans in forest

For some special forests, which we do not cut, i.e., do not supply timber, under legal, economic or specific environmental constraints, these forests include:

- Forests that have legal restrictions or other restrictions resulting from political decisions. These restrictions completely exclude or severely limit the supply of timber. Especially for environmental or biodiversity conservation reasons, e.g. nature reserve forests, mother lode forests, experimental forests.

- Forests located in less productive areas, where harvesting and transportation costs are too high, or where the quality of the wood is too low, are excluded from harvesting. Since such special cases are rare, the resulting forests that cannot be cut down are not considered in this paper.

We manage forests by cutting them at different harvesting rates Cut. For different kinds of forests, the management methods vary widely, and there are transition points between the management methods of each forest.

We need to develop appropriate management plans for the corresponding forest to ensure that its carbon sequestration is large while also ensuring that the other values of that forest are the most beneficial to humans. Then our decision variable is the cutting ratio Cut_s,and the objective function is:

$$\max \quad Value_{total} = \sum_{i=1}^{3} Rev_i, \tag{29}$$

s.t. eq(15)-(28).

5.4 Analysis of Result

With the model in the first problem, we classify forests by different harvesting possibilities into fast-growing forests (harvesting as much as possible), normal forests (harvesting within a reasonable cycle) and ecological forests (not harvesting as much as possible), and the trends of harvest rate and carbon sequestration for these three types of forests are shown in Figure 11 below. Immediately after, in Model 2, we consider the impact of more factors on harvesting management.

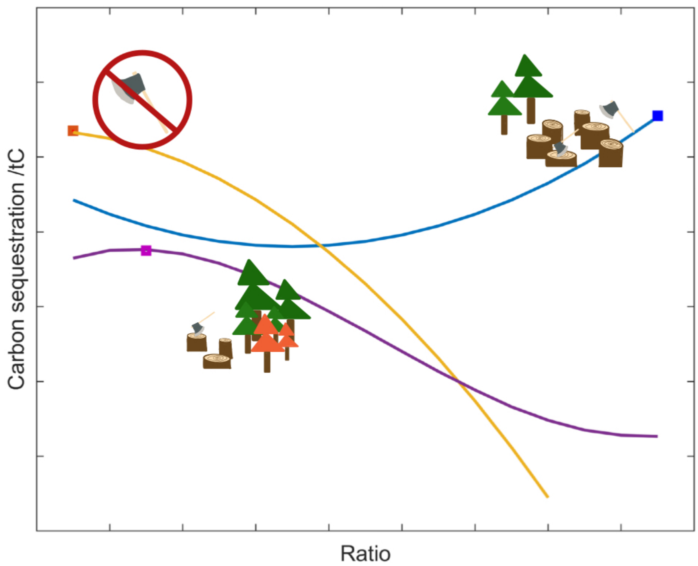

Figure 11: Diagram of our forest classification

Temperature changes brought about by different environments can have different effects on the growth of different trees. For example, since there is an optimum temperature for tree growth, temperature can affect the process of tree accumulation change, as shown in Figure 12, and thus affect the amount of carbon sequestered during tree growth. Also, because different trees have different adaptations to temperature intervals, their final stable accumulation impacts are also different, as shown in Fig. 13

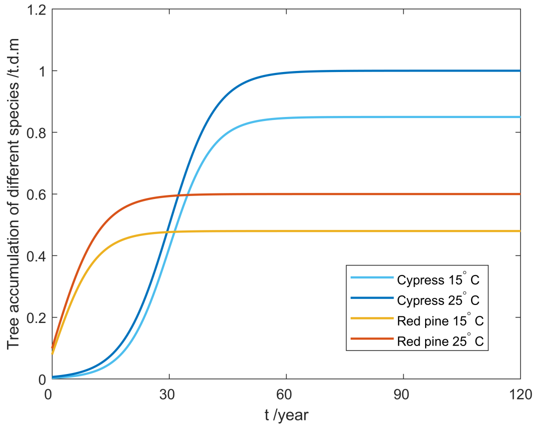

Figure 12: Influence of temperature on the process of change in So

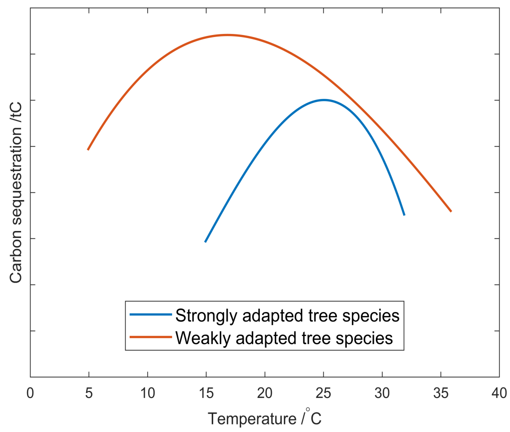

Figure 13: Temperature acclimatization of different trees and their maximum stable So

In Question 1, we have determined the optimal cutting time for different trees when only their own and forest product carbon sequestration is considered to be maximum. In this question, we consider other values of the forest in addition to carbon sequestration. These values indirectly affect the optimal cutting time of trees, thus reducing the originally obtained cutting rate Cut_s and causing the accumulation of trees cut at the right age. Ultimately, the stable optimal cutting time for trees will be longer than the optimal cutting time in the first question and the comparison between the two is shown in the figure 15 below.

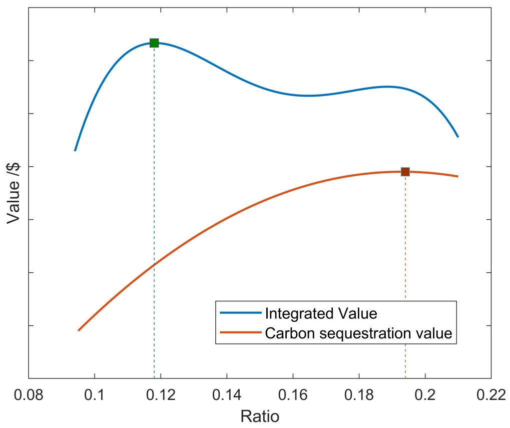

Figure 14: Only carbon sequestration and comprehensive consideration are compared

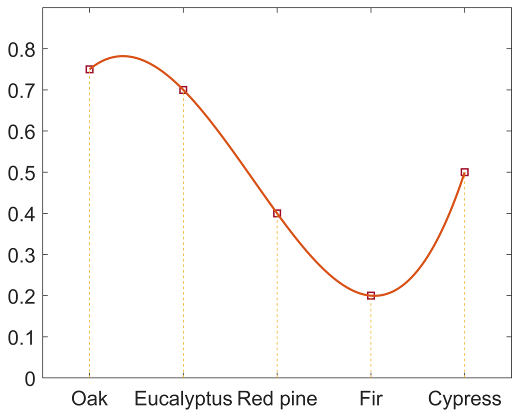

Figure 15: The rate of change at the optimum felling point for trees

At the same time, we found that there is variability in the impact of various influencing factors on the optimal felling time of different trees, shown as Figure **??**, for the faster growing tree species, the felling point to move back a larger proportion because of the demand of ecological stability, for example, oak trees were extended from 4 years to 7 years, an increase of 75%; similarly for trees with already long felling time, such as Cypress and various fruit trees, felling point to move back a larger proportion about 50% because of the demand of both economy and society.

Thus, based on the first question, we redefine the rotation period for the three major forest species: the cutting ratio of fast-growing forests is 10%-20%, and the rotation period is 5-10 years, such as firewood forests; the cutting ratio of normal forests is 2.5%-5%, and the rotation period is 20-40 years, such as timber forests; and the cutting ratio of ecological forests is less than 1.43%, and the rotation period is more than 70 years, such as non-wood product forests and shelter forests.

6 Model III: A case of specific forest management

Forests often have multiple uses.As shown in Figure 10, there are huge differences between forests. For example,forests for special purpose are often used for national defense and scientific experiments, which are explicitly forbidden to be felled, while others are allowed to cut down.Non-wood product forest means we get products from living trees, such as fruit trees and rubber trees, which should not be felled economically unless there is more profit. However,firewood forests are always being rotated for revenue.

We selected the Penobscot Experimental Forest (PEF) in the towns of Bradley and Addington, Maine, covering 3,855 acres, shown as Figure 16. In this forest, oak, pine, fir, and maple are the main dominant species, with 517031, 244509, 196395, and 140906 thousand trees, respectively, of which the first three have reached a total of 64.59%. In order to facilitate the presentation of results, we select the first three trees for calculation.

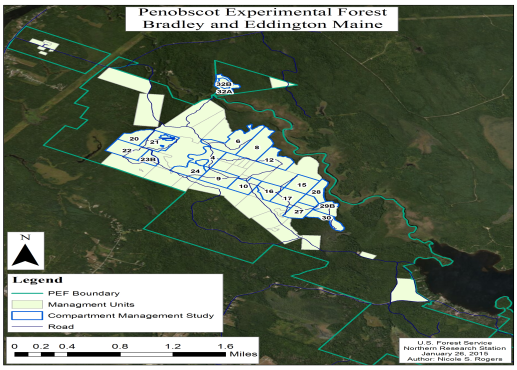

Figure 16: Penobscot Experimental Forest

Figure 17: Oka, Pine , Fir and Maple

6.1 The CO_2 sequestration in 100 years with the best forest management

Firstly, we calculate the optimal felling time for different tree species corresponding to different products by Figure 18(From Wikipedia [**4**, **5**]). And then,we bring it and the data of tree growth into the integrated model to find the optimal felling proportion of each species of tree with the maximum carbon sequestration under this plan.

The maximum carbon sequestration curve of this forest can be obtained by summing the maximum carbon sequestration of various trees mentioned above.Pay attention to the information of Figure 20, we find that the maximum carbon sequestration of this forest can be reached at a stable value of 3138.366tC with cutting rate of 4.8% per year.

Considering that the actual tree growth is an unpredictable process, the Monte Carlo algo-

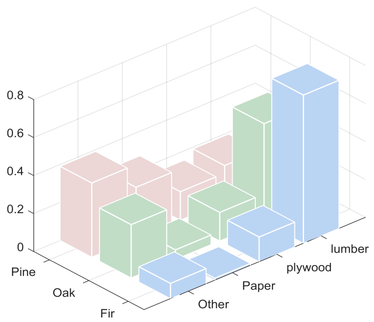

Figure 18: The proportion of different uses of trees

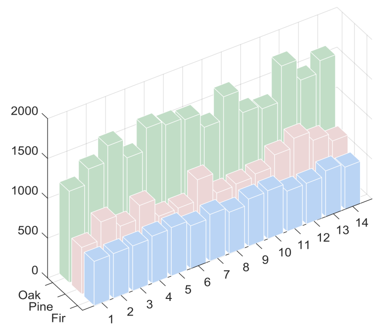

Figure 19: Carbon sequestration with felling rates

rithm is used to simulate the above results with some fluctuations to simulate the actual carbon sequestration condition during the 100-year accumulation process more closely.Final result is shown in Figure 21.

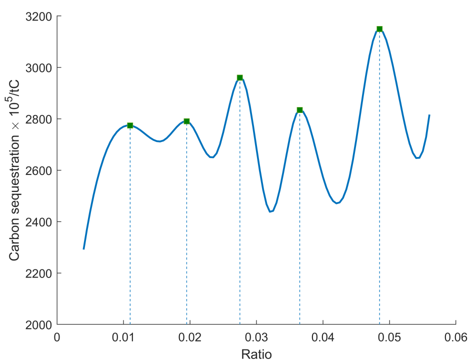

Figure 20: Specific forest carbon sequestration with felling rates

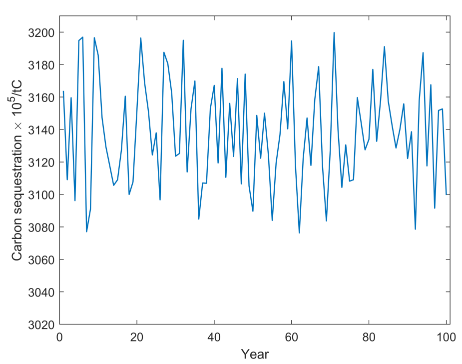

Figure 21: Centennial carbon sequestration curves of specific forests

In summary, these three trees only account for 64.59% of the forest.By extension, we can obtain the carbon sequestration of this experimental forest as $485847.26007tC$,and the CO_2 sequestration C_{CO2} as $1.78 \times 10^6 t$.

6.2 Timeline transition strategy

We can calculate the felling cycle based on the cutting rate. Assuming that we get an optimal management plan for the forest that is 10 years longer than the management plan currently applied for it, the optimal cutting rate is smaller than the cutting rate currently used. To make a smooth transition between the two forest management plans, we need to design a transition period strategy based on the needs of the forest managers and their users.

There are different transition strategies for different types of forests and their managers and users.

- Commercial Forests:

  Since the managers and users of commercial forests usually have absolute management rights over the commercial forests and can decide on the planting of commercial forests, it is possible to consider replanting a short-growing, fast-growing and high-yielding forest species after cutting. In addition, when replanting a new forest species, consider intercropping to develop a three-dimensional structure of this type of forest.

- Non-commercial Forest:

  (1) Shelter Forest:For some shelter forests that can be managed for cutting, their protective role needs to be maintained. When the optimal cut cycle is longer than the current cut cycle, it means that the current cutting rate is higher than the optimal cutting rate.At the same time, the protective function of the forest has been damaged to a certain extent. In this regard, the manager needs to compensate for the protective effect of the shelter forest by considering an application to expand the area of the shelter forest and plant trees with the original protective effect of the forest in order to repair its protective effect.

  (2) Forest for Special Purpose: For some forests that can be cut for special purpose, such as experimental forests, their large cutting rate may cause a shortage of age-appropriate experimental trees at a later stage. In this regard, we can consider expanding their planting area in an appropriate amount and purchasing corresponding materials when there is a shortage of experimental materials. At the end of the transition period, we can consider selling the trees on the expanded area compared to the area before the transition period, so that the original area of the forest can be restored and the cost of purchasing the experimental material can be subsidized at the same time.

7 Model Analysis and Sensitivity Analysis

7.1 The Influence of Different tree initial age

Definitely, the initial age of the forest tree species is independent of the maximum carbon sequestration in the cycle iteration, as shown in Figure 22. We test the carbon sequestration curve from 0 to 3 times the optimal felling time for a certain tree species producing a certain product and see that the carbon sequestration is a definite value when the cycle is long enough.

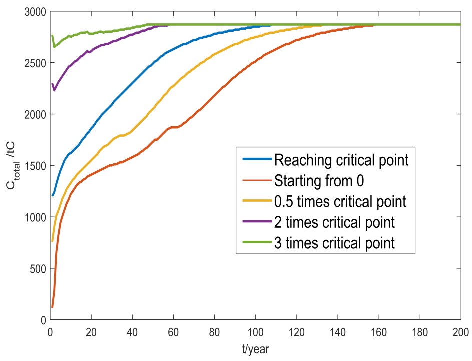

Figure 22: Effect of tree initial age on carbon sequestration curve

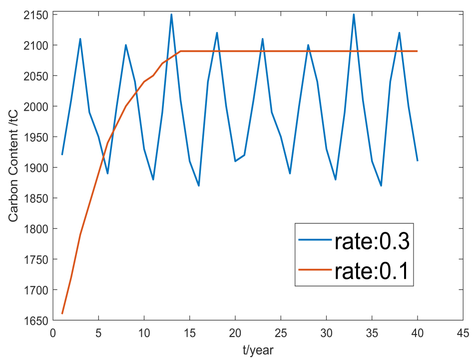

Figure 23: Carbon sequestration curves for specific forests at a rate of 0.3 versus 0.5

7.2 Different proportion of selective forest cutting

When traversing the cutting rate, we find that if the cutting rate is too large, the newly planted trees will be cut before they reach the optimal cut point. We choose to wait until they reach the optimal cutting time before cutting them. Thus, when the overall rate of cutting is too fast, there are persistent fluctuations in the final carbon sequestration curve.Figure 23 shows the change in the carbon sequestration curve due to a change in the cutting rate of a single tree species.

Therefore, we choose the stable point of the carbon sequestration curve as the maximum cutting rate. Therefore, for a single continuous cut forest management model (such as fast growing forest), the maximum annual cut rate is the inverse of the felling cycle.

8 Strength and Weakness

8.1 Strength

- We analyze the problem based on logistic equation and IPCC method,and consider the influence of various factors on forest carbon sequestration and its integrated evaluation model from several perspectives,so that the model we established is of great validity.
- Our model is fairly robust due to our careful corrections in consideration of real-life situations and detailed sensitivity analysis.
- Via Python code, we simulate the carbon sequestration curves of multiple trees and their products. The outcome is vivid for us to understand the changing process,and the result is easy to reproduce.
- Besides, Our model is built based on tree and product species, and can be applied to a variety of different forests with good generalizability.

8.2 Weakness

- The model is relatively complex, and when there are too many species, the solution of the model parameters may be difficult. However, the results of the solution are more interpretable.
- We do not consider the impact of major natural disasters on forests. However, we simulate the fluctuations of the general situation, and the model still reflects the reality better in the general case.

9 Further Discussion

In model I, Monte Carlo method is used to simulate the loss rate of raw wood during transportation and processing from raw wood to product. However, the loss rate of the one product widely among different manufacturers.

In this regard, stochastic frontier analysis can be used to curve fit the loss rate of the configuration information of different processing plants, and then the corresponding processing plants can be matched according to the tree species and product proportion of different forests. Finally, the real loss rate of wood products of the whole forest can be obtained.

References

[1] Gorte R W. Carbon sequestration in forests[M]. DIANE Publishing, 2009.

[2] Smith, James E.; Heath, Linda S.; Skog, Kenneth E.; Birdsey, Richard A. 2006. Methods for calculating forest ecosystem and harvested carbon with standard estimates for forest types of the United States. Gen. Tech. Rep. NE-343. Newtown Square, PA: U.S. Department of Agriculture, Forest Service, Northeastern Research Station. 216 p.

[3] Zhao RJ, Jiang ZW, Fei BH, Wang XM. Current status of eucalyptus wood processing and utilization research in Australia[J]. World Forestry Research,2003(03):58-61.DOI:10.13348/j.cnki.sjlyyj.2003.03.012.

[4] https://en.wikipedia.org/wiki/Oak

[5] https://en.wikipedia.org/wiki/Pinus_resinosa

[6] Wu Shuhong, Zhu Jianhua, He Youjun. Guidelines for measuring carbon stocks in forest ecosystems [S]. State Forestry and Grassland Administration. 2019.

[7] Lai, Yongjian. Productivity analysis of Chinese wood processing and products firms–based on stochastic frontier model[J]. Forestry Economic Issues,2014,34(01):72-77.DOI:10.16832/j.cnki.1005-9709.2014.01.013.

[8] FAO and UNEP. The State of the World's Forests 2020: Forests, Biodiversity and People. Rome.2020:65.

[9] Caparros A, Campos P, Martin D. Influence of carbon dioxide abatement and recreational services on optimal forest rotation[J]. International Journal of Sustainable Development, 2003, 6(3): 345-358.

Cutting to be Better

Forest and carbon sequestration

Introduction

Severe climate change in recent years has brought renewed attention to greenhouse gases. To mitigate the effects of climate change, we can use photosynthesis, a process called "carbon sequestration" to sequester carbon dioxide from the atmosphere in plants, soil and water. Therefore, forests made of trees play an important role in mitigating climate change.

Selective cutting

Some people believe we should never cut down any trees, however, natural forest is high density and canopy density, a considerable part of the growth is poor, strength is weak while diseases and insect pests is serious. Without selective cutting, this part of the forest will inevitably come to an end.

Selective cutting is to cut trees that are mature or should be cut in a certain period of time, and to keep immature or unsuitable trees on the forest land, so that the forest after cutting still maintain trees of all ages. In fact, moderate deforestation can have a positive impact on ecology, economy and society at the same time.

Ecology

Selective cutting allows the forest to continuously achieve local regeneration with the ground always remaining covered with trees, thus forming a heterogeneous complex forest, promoting the distribution of species structure and contributing to the ecological diversity of the forest. Selective cutting can convert forest trees of appropriate age into forest products. The forest products are also capable of carbon sequestration. Choosing an appropriate rate of selective cutting can increase the overall carbon dioxide sequestration of the forest and its ancillary products, which can mitigate the greenhouse effect to a certain extent and have a positive ecological impact.

Economy

As a renewable resource, appropriate logging of forests can make full use of the resources for economic effect and bring some income to forest managers, which can compensate for the expenses of managing forests and form a virtuous circle.

Society

The human labor required in the process of trees from planting to cutting to becoming tree products fully drives the wood industry, bringing jobs and moral support to tens of millions of people.

Selective cutting shall be strictly carried out

During selective cutting, technical indicators will be strictly controlled to ensure the positive succession of natural forests, and the overall coordination will be emphasized to maintain reasonable density and biodiversity. In particular, in horizontal and vertical structure, more attention should be paid to the reasonable collocation of tree layer, shrub layer and herb layer, so as to ensure the forest vegetation coverage, species mix well and avoid competition for nutritional as even as possible.

Absolutely, rigorous selective cutting process will bring positive benefits to us.

Summary

Therefore, the importance of selective cutting management cannot be overstated. Knowing about its positive impact on economic, ecological and social, we hope forest managers' management plans for forest should include harvesting management rather than leaving it untouched.